CHAPTER IV

ALGORITHMS FOR BUILDING HEAT GAIN AND HEAT LOSS CALCULATIONS

In this chapter, the algorithms for calculating hourly heat flow through building envelopes were discussed. For calculating transient heat flow through opaque building materials having considerable thermal capacity, Heat Transfer Function introduced by ASHRAE[1] was studied. This method was based on Thermal Response Factor Method developed by D.G. Stephenson and G.P. Mitalas in 1967. The method permits the utilization of hourly values of weather data for a whole year.

Conventional "$UA(T_i-T_o)$" method was adopted for calculating conductive heat flow through materials having no significant thermal capacity such as windows and doors.

These algorithms were applied in Chapter VI to develop a computer model for calculating building heat gain and loss on hourly basis.

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[1] ASHRAE Handbook of Fundamentals, 1976

4.1 Thermal Response Factor Method

Thermal Response Factor Method was first introduced by Mitalas and Stephenson[2] in 1967. This is a way of presenting data on the dynamic thermal characteristics of a room for air-conditioning design calculations by digital computers.

Response factors can only be used, however, if the system can be represented by linear and invariable equations. Mitalas[3] has shown that linear equations are adequate to describe the heat transfer processes that occur in an air conditioned room. Linearity implies that the magnitude of the response is linearly related to the magnitude of the excitation; invariability means that equal excitations applied at different times always produce equal responses.

The response of a linear, invariable system to a unit time-series excitation function (1,0,0,...) is called the unit response function, and the time-series representation of this unit response function is the set of response factors (Stephenson D.G. and Mitalas G.P., 1967).

For example, the excitations could be radiant energy from lights of sun that is absorbed at any surface, or sol-air temperature for the outside surface of outer wall. Figure 4.1 shows an example of response function due to solar radiation incidence on the floor.

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[2] G.P. Mitalas and D.G. Stephenson: "Room Thermal Response Factors", ASHRAE Transactions, Vol. 73, Part I, 1967

[3] Mitalas, G. P.: "An Assessment of Common Assumptions in Estimating Cooling and Space Temperatures", ASHRAE Transactions, Vol. 71, Part II, p. 72, 1965.

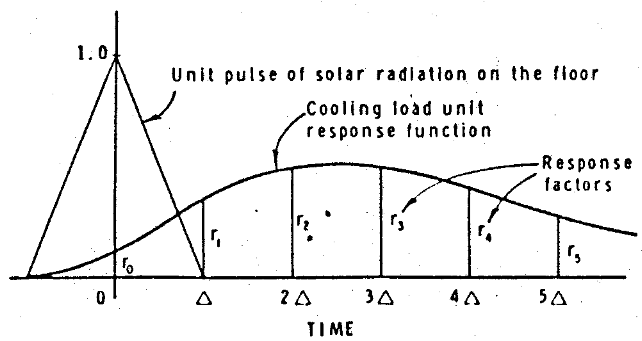

Fig. 4.1 Unit excitation and unit response functions (Stephenson, D. G. and Mitalas G. P., 1967)

The response factor method is convenient for room thermal performance calculations because it requires only that the excitation functions be expressed as time-series, and it shows very clearly the influence of each excitation of the final result. The response factors need to be determined only once for a particular room. They can then be combined with the appropriate excitation functions to obtain the response in a specific case. The ease of separating the room characteristics from the characteristics of the external environment is one of the major advantages of the method, for it permits the separate calculation of building and environment data.

ASHRAE adopted this method for hourly building cooling load calculation and tabulated the Heat Transfer Functions of various kinds of typical opaque materials for roofs, external walls, and internal walls and ceilings.

4.2 Conductive Heat Flow through Opaque Materials

The basis for this calculation sequence lies in the relationship:

$$Qcon = A\sum_{n=0}^{m} b_n\, Ts_{t-ndt} - \sum_{n=1}^{m} d_n\, Qcon_{t-ndt} - A\, T_i \sum_{n=0}^{m} c_n \qquad \text{(Eq. 4.1)}$$

where $Qcon$ = heat gain/loss due to conduction at time t.

$Qcon_{t-ndt}$ = the heat gain/loss at time t-ndt.

Ts_{t-ndt} = hourly out-door sol-air temperatures for the particular surface.

A = surface area.

The equation assumes that indoor air temperature T_i is constant. The constants b_n, d_n, and $\sum c_n$ are the thermal transfer function coefficients which are tabulated in the ASHRAE Handbook of Fundamentals (1977). The value of m depends on the material's thermal capacity. This equation determines the heat gain/loss by conduction through any opaque surface that has any significant thermal lag (W. Murphy, 1986). As this equation needs thermal history of as much as 6 hours earlier than the time of current calculation, the weather data of up to last 6 hours' of the previous month are to be input for the calculation of the conductive heat gain/loss at first few hours of current month.

4.3 Heat Flow through Window

The heat flow through window glass can be described as shown in Figure 4.2, and the total heat gain or loss, Q_{win}, can be expressed by:

$$Q_{win} = \text{(radiation transmitted through the glass)} + \text{(inward/outward flow of absorbed solar radiation)} + \text{(heat flow due to outdoor-indoor temperature difference)}$$

In this equation each term is to be calculated separately and then combined to determine the total amount of heat flow through windows.

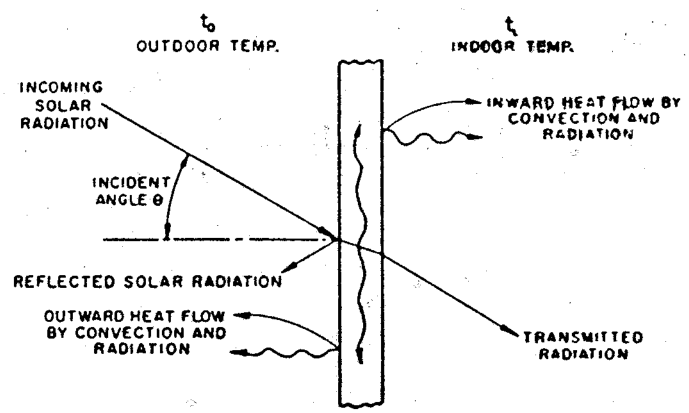

Fig. 4.2 Heat flow through window glass (ASHRAE, 1985, p. 27.18)

4.3.1 Glass Transmission, Absorption and Reflection

For opaque surfaces, the sum of absorptance and reflectance must be unity. If the surface is transparent to the incident radiation to any degree, the sum of absorptance, reflectance, and transmittance must be unity (i.e., the incident radiation is absorbed, reflected, or transmitted). This relationship holds for surfaces, or layers of material of finite thickness. Transmittance, like reflectance and absorptance, is a function of wavelength, angle of incidence of the incoming radiation, the refraction index, and the extinction coefficient of the material (John A. Duffie, et al, 1974, p. 108).

With the given data of glass thickness, L, refraction index, n, extinction coefficient, k, the transmission, absorption and reflection coefficients of the glass for different solar radiation incident angles can be calculated. The extinction coefficient, k, is assumed to be a constant in the solar spectrum. For glass, the value of k varies from about 0.04/cm for "water white" glass to about 0.32/cm for poor glass having greenish cast of edge (Duffie, John A., et al, 1974, p. 112).

With the solar radiation incident angle, i, on a window surface, the refraction angle, r, is given by:

$$r = \sin^{-1} \frac{\sin i}{n} \qquad \text{(Eq. 4.2)}$$

Then, reflection ratio, σ, was derived by Fresnel and given by:

$$\sigma = \frac{1}{2} \left[\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{\tan^2(i-r)}{\tan^2(i+r)} \right] \qquad \text{(Eq. 4.3)}$$

And transmission ratio of the radiation through a distance x is given by:

$$\tau(x) = e^{-kx} \qquad \text{(Eq. 4.4)}$$

where $x = L/\cos(i)$.

Using the values from equations 4.2, 4.3 and 4.4, the transmission (T), reflection (R) and absorption (A) coefficients can be calculated as follows:

$$T = \frac{\tau(x)(1 - r)^2}{1 - \tau(x)\, r^2} \qquad \text{(Eq. 4.5)}$$

$$R = r\left[1 + \frac{\tau(x)\,(1-r)^2}{1-\tau^2(x)\, r^2}\right] \qquad \text{(Eq. 4.6)}$$

$$A = 1 - R - T \qquad \text{(Eq. 4.7)}$$

As shown in figure 4.3 the transmission declines sharply at incident angles greater than 50°.

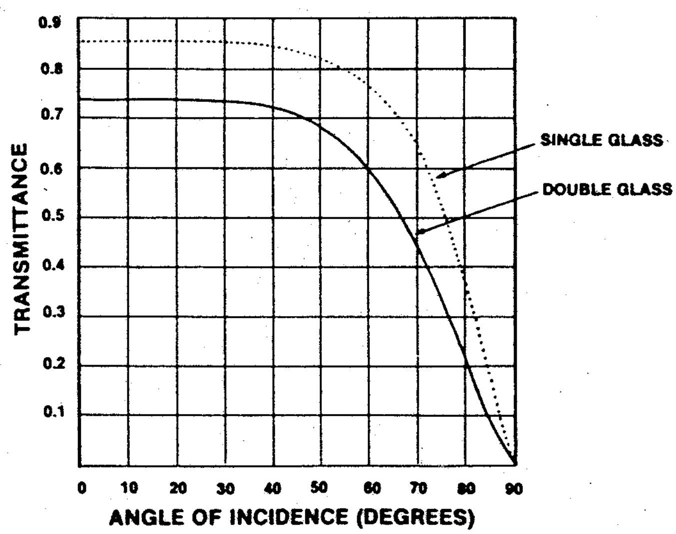

Fig. 4.3 Glass Transmission
(Mazria, 1979, p.20)

4.3.2 Radiative Heat Gain Through Window

The radiation transmitted through window glass, Qr, is given by the total radiation incident on the windows and the transmission coefficients of the glass materials.

$$Qr = T\ I_{TV} \qquad \text{(Eq. 4.8)}$$

where I_{TV} = total radiation incident on the window glass.

4.3.3 Conductive Heat Flow through Window

Conductive heat gain/loss for the window is found by conventional UAdT because of negligible heat capacity of the glass materials, where the dT is simply the instantaneous inside to outside temperature difference.

$$Qc = U\ A\ (T_o - T_i) \qquad \text{(Eq. 4.9)}$$

where Qc = conductive heat gain or loss through glass

U = heat transmission coefficient of the glass

A = area of the glass

T_o = outdoor temperature

T_i = indoor temperature

4.4 Solar Radiation on Outdoor Surfaces

4.4.1 Direct Radiation On Walls

For any surface, the incident angle ϕ is related to α, r and the tilt angle Σ of the surface by: (ASHRAE 1985, p. 27.3)

$$\cos\phi = \cos\alpha \cos r \sin\Sigma + \sin\alpha \cos\Sigma \qquad \text{(Eq. 4.10)}$$

where α = solar altitude angle

r = wall-solar azimuth angle

Σ = surface tilt angle

For a horizontal surface like flat roof with $\Sigma = 0°$, above equation becomes:

$$\cos\phi = \sin\alpha \qquad \text{(Eq. 4.11)}$$

Direct radiation on a horizontal surface I_{DH} is given by:

$$\begin{aligned} I_{DH} &= I_{DN} \cos\phi \\ &= I_{DN} \sin\alpha \qquad \text{(Eq. 4.12)} \end{aligned}$$

where I_{DH} = direct solar radiation on a horizontal surface

I_{DN} = direct solar radiation on a normal surface.

For a vertical surface such as a wall with $\Sigma = 90°$, equation 4.10 becomes:

$$\cos\phi = \cos\alpha \cos r \qquad \text{(Eq. 4.13)}$$

And direct radiation on a vertical surface I_{DV} is given by:

$$I_{DV} = I_{DN} \cos \phi \qquad \text{(Eq. 4.14)}$$

where I_{DV} = direct solar radiation on a vertical surface.

4.4.2 Diffuse Radiation On Walls

The diffuse radiation falling on any surface consists of radiation from the sky and part of the reflected solar radiation from adjacent surfaces, particularly the ground lying south of the surface in question. (ASHRAE 1985, p. 27.8)

A simplified general relation for the diffuse solar radiation (I_{sv}) that falls on a vertical surface from sky is given approximately by:

$$I_{sv} = .5\ I_{dh} \qquad \text{(Eq. 4.15)}$$

where I_{dh} = sky diffuse radiation.

The radiation I_{gr} reflected from adjacent ground to a vertical wall is given approximately by:

$$I_{gr} = .5\ I_{TH}\ \sigma \qquad \text{(Eq. 4.16)}$$

where I_{TH} = total solar radiation on a horizontal surface

σ = ground reflection coefficient.

The total diffuse radiation (I_{dv}) on a vertical surface is given by the sum of diffuse radiation from the sky and that from the ground.

$$I_{dv} = I_{sv} + I_{gr} \qquad \text{(Eq. 4.17)}$$

4.4.3 Total Radiation On Walls

Total radiation I_{TV} falling on a vertical surface is given by:

$$I_{TV} = I_{DV} + I_{dv} + I_{gr} \qquad \text{(Eq. 4.18)}$$

4.5 Sol-Air Temperature

Sol-air temperature (T_s) is a convenient way of accounting for solar radiation effects on the wall or roof. It is expressed by:

$$T_s = T_o + (\alpha_s/h_o)\ I_{TOT} - (\varepsilon_o/h_o)\ dR \qquad \text{(Eq. 4.19)}$$

where T_o = outdoor temperature

I_{TOT} = total radiation on a surface

α_s = solar absorptivity of outdoor surface

h_o = outdoor film coefficient.

The last term in Equation 4.19 is a correction to the infrared radiation exchange already incorporated in the film coefficient h_o (W. Murphy, 1986). For approximation of this term, 7°F for horizontal surface and 0°F for vertical surface may be used (ASHRAE 1977, p. 25.4). Also, in stead of calculating the coefficient α_s/h_o, 0.15 and 0.30 may be used for light-colored surface and dark-colored surface, respectively (ASHRAE 1977, p. 25.4).